

An anisotropic cosmological solution to the Maxwell- $Y(R)$ gravity

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04 June 2012, *file YRanisotropic01.tex*

Abstract

In this paper, we investigate an anisotropic model of the universe by using the modified gravity approach which involves the non-minimal $Y(R)F^2$ -type couplings of electromagnetic fields to gravity. After we derive field equations by a first order variational principle from the lagrangian of non-minimally coupled theory, we look for a spatially flat cosmological solution with a large-scale magnetic field and correspondingly some kind of anisotropy. At the end we estimate certain values according to observations for three parameters occurring in the solutions.

PACS numbers: 04.50.Kd, 98.80.Jk

Keywords: Acceleration of universe, modified gravity, magnetic field

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1 Introduction

The inflation and the late-time acceleration of the universe [1]-[7] are two most exciting observations in cosmology. Besides, in spite of that modern cosmology is based on the cosmological principle which assumes that the universe is homogeneous and isotropic, there are some reports in the literature announcing some detection of anisotropy on spatial scales where dark energy becomes important [8]-[10]. There exist various scenarios to explain these observations [11]-[21]. In this paper, we consider the modified gravity approach which involves a coupling between gravity and electromagnetism in $Y(R)F^2$ form to explain them for the following reasons:

- The non-minimal models allow us to find the solutions satisfying solar system and cosmological tests for some coupling parameter values [22]-[24]. They are the modified dynamical models in which the modification of gravity is generated from the electromagnetic fields and the vice versa is also valid.
- The non-minimal couplings with RF^2 -type which break the conformal invariance of electromagnetic field were investigated in various aspects [25]-[36]. It is remarkable that they can arise from the calculation of QED one-loop vacuum polarization on a curved background [27] and from the effective renormalization-group improved theory [37]. Then, they lead to electromagnetic quantum fluctuations at the inflationary stage, which lead the inflation [38]-[42]. Because of the inflation at that time, the scale of the electromagnetic quantum fluctuations can be stretched towards outside the Hubble horizon and they give rise to classical fluctuations. Thus, they can be the reason of the large scale magnetic fields observed in clusters of galaxies [37],[43]-[46].
- Such models, $f(R)$ -Maxwell [47]-[48], $f(G)$ -Maxwell [49], $f(R)$ -Yang-Mills [37] and $f(G)$ -Yang-Mills gravity [50] are considered to explain both inflation and the late-time acceleration of the universe.

Although there are many other theories of modified gravity which involve a function of curvature scalar the non-minimal couplings with electromagnetic fields need more investigation to appear new insights on gravity, electromagnetism and/or electromagnetic duality. Also, it is important to compare consistency of the non-minimal theories with the other models according to observations. Especially, to find cosmological solutions with magnetic fields can be important for the origin of cosmic magnetic fields in the universe. In this paper we consider the non-minimal couplings of gravity with electromagnetic fields by using the method of Lagrange multipliers and the algebra of exterior differential forms. Then we investigate some anisotropic cosmological solutions and discuss our results.

2 The Theory of $Y(R)F^2$ Gravity

We will derive our field equations of the non-minimal theory by a variational principle from the action

$$I[e^a, \omega^a_b, F] = \int_M L = \int_M \mathcal{L} * 1, \quad (1)$$

where e^a are the orthonormal basis 1-forms, $\omega_{ab} = -\omega_{ba}$ are the Levi-Civita connection 1-forms, F is the electromagnetic field 2-form and M is the four-dimensional differentiable manifold endowed with the metric $g = \eta_{ab}e^a \otimes e^b$, $\eta_{ab} = \text{diag}(-+++)$. The orientation of the manifold is fixed by the Hodge map $*1 = e^0 \wedge e^1 \wedge e^2 \wedge e^3$. The torsion 2-forms, T^a , and the curvature 2-forms, R^a_b , of the spacetime are given by the Cartan-Maurer structure equations

$$T^a = de^a + \omega^a_b \wedge e^b, \quad (2)$$

$$R^a_b = d\omega^a_b + \omega^a_c \wedge \omega^c_b. \quad (3)$$

We consider the following lagrangian density 4-form [37, 22]

$$L = \frac{1}{2\kappa^2} R * 1 - \frac{1}{2} Y(R) F \wedge *F + \lambda_a \wedge T^a + \mu \wedge dF, \quad (4)$$

where $\kappa^2 = 8\pi G$ is the Newton's universal gravitational constant ($c = 1$), R is the curvature scalar, Y is the non-minimal coupling function of R , λ_a and μ respectively Lagrange multiplier 2-forms constraining torsion to zero ($T^a = 0$) and electromagnetic fields to the homogeneous case ($dF = 0$, then we can consider $F = dA$).

The non-minimal gravitational coupling of the electromagnetic field, $Y(R) \neq 1$, may change the value of the fine structure constant, i.e. the strength of the electromagnetic coupling. Accordingly, some observations such as radio and optical quasar absorption lines, the anisotropy of the cosmic microwave background radiation and big bang nucleosynthesis etc (for a recent review, see [51]) can give some constraints on the deviation of the non-minimal electromagnetism from the minimal one.

The electromagnetic field components are read from the expansion $F = \frac{1}{2} F_{ab} e^a \wedge e^b$. We use the shorthand notations: $e^a \wedge e^b \wedge \dots = e^{ab\dots}$, $\iota_a F = F_a$, $\iota_{ba} F = F_{ab}$, $\iota_a R^a_b = R_b$, $\iota_{ba} R^{ab} = R$ where ι denotes the interior product such that $\iota_b e^a = \delta^a_b$. The field equations are obtained via the independent variations of the action with respect to $\{e^a\}$, $\{\omega^a_b\}$ and $\{F\}$.

The infinitesimal variations of the total Lagrangian density L (modulo a closed form) are given by

$$\begin{aligned} \delta L = & \frac{1}{2\kappa^2} \delta e^a \wedge R^{bc} \wedge *e_{abc} + \delta e^a \wedge \frac{1}{2} Y(R) (\iota_a F \wedge *F - F \wedge \iota_a *F) + \delta e^a \wedge D\lambda_a \\ & + \delta e^a \wedge Y_R (\iota_a R^b) \iota_b (F \wedge *F) + \frac{1}{2} \delta \omega_{ab} \wedge (e^b \wedge \lambda^a - e^a \wedge \lambda^b) \\ & \delta \omega_{ab} \wedge \Sigma^{ab} - \delta F \wedge Y(R) *F + \delta \lambda_a \wedge T^a + \delta F \wedge d\mu + \delta \mu \wedge dF \end{aligned} \quad (5)$$

where $Y_R = \frac{dY}{dR}$ and the angular momentum tensor

$$\Sigma^{ab} = \frac{1}{2}D[\iota^{ab}(Y_R F \wedge *F)]. \quad (6)$$

The Lagrange multiplier 2-forms λ_a are solved uniquely from the connection variation equations

$$e_a \wedge \lambda_b - e_b \wedge \lambda_a = 2\Sigma_{ab}, \quad (7)$$

by applying the interior product operator twice

$$\lambda^a = 2\iota_b \Sigma^{ab} + \frac{1}{2}(\iota_{bc} \Sigma^{bc}) \wedge e^a. \quad (8)$$

We firstly substitute the λ_a 's into the δe^a equations, then perform some simplifications and finally we arrive at the modified Einstein's equation

$$\begin{aligned} \frac{1}{2\kappa^2}R^{bc} \wedge *e_{abc} + \frac{1}{2}Y(\iota_a F \wedge *F - F \wedge \iota_a *F) + Y_R(\iota_a R^b) \iota_b (F \wedge *F) \\ + \frac{1}{2}D[\iota^b d(Y_R F_{mn} F^{mn})] \wedge *e_{ab} = 0, \end{aligned} \quad (9)$$

while the modified Maxwell equations read

$$dF = 0, \quad d(Y * F) = 0. \quad (10)$$

3 Cosmological Solutions

We will be considering only a magnetic field. Thus expectedly the direction, defined by it, is not same as the others. Accordingly, instead of an isotropic universe we naturally investigate cosmological model with some kind of anisotropy. The latter can be caused by electromagnetic field like here, or rotation, or is directly connected with nontrivial shear of a metric. Thus we look for exact solutions in the form of the anisotropic metric

$$g = -dt^2 + a(t)^2 dx^2 + b(t)^2 (dy^2 + dz^2) \quad (11)$$

where $a(t)$ is the polar expansion function and $b(t)$ is the equatorial expansion function. We also think of the case in which magnetic fields are mainly generated rather than electric fields because we are interested in the generation of large-scale magnetic fields

$$F = B(t)e^{23} \quad (12)$$

where $e^0 = dt$ and $e^1 = a(t)dx$, $e^2 = b(t)dy$, $e^3 = b(t)dz$ are the orthonormal basis 1-forms and $B(t)$ represents the time-dependent x -component of magnetic field. This situation is realized if $Y(R)$ increases rapidly in time during inflation [47]. This is the case here, see Fig.3.b. Besides, we note one point more. In the present model, the large-scale magnetic fields may be generated

due to the breaking of the conformal invariance of the electromagnetic field through a coupling with the scalar curvature, $Y(R)F \wedge *F$.

As the homogenous Maxwell equation (10i) gives us

$$B(t) = B_0/b^2(t) \quad (13)$$

where B_0 is a constant, the inhomogeneous Maxwell equation (10ii) is satisfied automatically. On the other hand, the modified Einstein equation (9) yields a set of complicated differential equations. In order to find some classes of exact solutions we firstly reduce them to a much simpler form by taking into account the following assumption

$$Y_R B^2 = -\frac{1}{\kappa^2}. \quad (14)$$

Thus we arrive at the equations

$$a_t b_t - a b_{tt} = 0, \quad \frac{a_{tt}}{a} - \frac{b_t^2}{b^2} + \frac{\kappa^2 B_0^2 Y}{2 b^4} = 0, \quad (15)$$

where subindex t denotes derivative with respect to cosmic time. The first one gives a relation between the polar and equatorial scale functions as

$$a = a_0 b_t \quad (16)$$

where a_0 arbitrary constant. Thus we obtain

$$2b^4 b_{ttt} - 2b^2 b_t^3 + \kappa^2 B_0^2 Y b_t = 0. \quad (17)$$

Although generally power-law expansion or exponential expansion is considered separately, here we investigate a solution putting those together in a multiplicative way

$$b(t) = t^n \exp(\beta t^m) \quad (18)$$

where n, m, β are arbitrary parameters. As comparing our results with observational data we use the mean expansion function

$$s(t) = (ab^2)^{1/3}. \quad (19)$$

Now we will try to determine n, m, β parameters by using the observational results of the first, second and third derivatives of the mean scale factor with respect to cosmic time, namely the present-day value of the Hubble parameter H_0 , the deceleration parameter q_0 and the cosmic jerk j_0 . Their definitions are given respectively by

$$H = \frac{s_t}{s}, \quad q = -\frac{1}{H^2} \left(\frac{s_{tt}}{s} \right), \quad j = \frac{1}{H^3} \left(\frac{s_{ttt}}{s} \right). \quad (20)$$

We can take the approximate present-day ($t = t_0 \approx 13.7$ Gyr) values for the concerned quantities as $H_0 \approx 70 \text{ km(s.Mpc)}^{-1} \approx 7.4 \times 10^{-2} \text{ Gyr}^{-1}$, $q_0 \approx -0.81$ and $j_0 \approx 2.16$ (or $-0.5 \leq j_0 \leq 3.9$)

[52],[53]. In accordance with observed data we estimate the observational values as $n \approx 1.063$, $\beta \approx 0.0000172$ and $m \approx 3.053$. We notice the smallness of β compared with others. We also depict the six graphs of Y, R, B, H, q vs cosmic time t and Y vs curvature scalar R for these values of n, m, β by setting $B_0 = 1$ and $\kappa = 1$, see Figs 1-3.

4 Conclusion

We have investigated consistently the early inflation, the late-time acceleration and the anisotropy of the universe for the non-minimally coupled gravitation theory with electromagnetic fields. After casting our model by a lagrangian 4-form we obtained the variational field equations. Then we found a class of solution under the assumption of a spatially flat anisotropic spacetime and a large-scale magnetic field which is generated due to the breaking of the conformal invariance of the electromagnetic field through its non-minimal gravitational coupling.

Thus we point out that the non-minimal gravitational coupling of the electromagnetic field and the generation of magnetic fields may be a source of the inflation of the early universe and of the late-time cosmic speed-up. Moreover, we give certain estimations on the related parameters in order that our model would exhibit a behavior consistent with the current understanding of the observed universe. Finally we give the six related graphs from which we deduce some information. (i) With help of computer, R dependence of Y is obtained via Fig.1.a and Fig.1.b as $Y(R) \approx -1/(0.2R^{1.1} - R^{1.6})$, see Fig.3.b. (ii) At the beginning the universe was too curved, but today its curvature is so tiny and we live in almost the de Sitter universe, Fig.1.b. (iii) As magnetic field was very dense for a short period in very far past, it is highly weak for a long time, Fig.2.a. This is consistent with observational data because its observational order of magnitude is micro Gauss for the scales of 500 kpc [45]. (iv) The behavior of the Hubble parameter is similar to the magnetic field's behavior, Fig.2.b. (v) The transition from cosmic speed-down to cosmic speed-up started at time $t \approx 8.2$ Gyr, Fig.3.a.

References

- [1] D. N. Spergel *et al.* [WMAP Collaboration], *Astrophys. J. Suppl.* **148** 175 (2003).
- [2] H. V. Peiris *et al.* [WMAP Collaboration], *Astrophys. J. Suppl.* **148** 213 (2003).
- [3] D. N. Spergel *et al.* [WMAP Collabora- tion], *Astrophys. J. Suppl.* **170** 377 (2007).
- [4] S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], *Astrophys. J.* **517** 565 (1999).

- [5] A. G. Riess *et al.* [Supernova Search Team Collaboration], *Astron. J.* **116** 1009 (1998).
- [6] P. Astier *et al.* [The SNLS Collaboration], *Astron. Astrophys.* **447** 31 (2006).
- [7] A. G. Riess *et al.*, *Astrophys.J.* **659** 98 (2007).
- [8] A. Antoniou, L. Perivolaropoulos, *JCAP* **12** 012 (2010).
- [9] R. Cooke, D. Lynden-Bell, *Mon. Not. Roy. Astron. Soc.* **401** 1409 (2010).
- [10] J. Colin *et al.* *Mon. Not. Roy. Astron. Soc.* **414** 264 (2011).
- [11] P. J. E. Peebles, B. Ratra, *Rev. Mod. Phys.* **75** 559 (2003).
- [12] T. Padmanabhan, *Phys. Rept.* **380** 235 (2003).
- [13] A. Mazumdar, J. Rocher, *Phys. Rept.* **497** 85 (2011).
- [14] Ö. Akarsu, T. Dereli, *Int. J. Theor. Phys.* **51** 612 (2012).
- [15] Ö. Akarsu, T. Dereli, "A Four-Dimensional Λ CDM-Type Cosmological Model Induced from Higher Dimensions Using a Kinematical Constraint", arXiv:1201.4545.
- [16] E. J. Copeland, M. Sami, S. Tsujikawa, *Int. J. Mod. Phys. D* **15** 1753 (2006).
- [17] R. Durrer, R. Maartens, *Gen. Rel. Grav.* **40** 301 (2008).
- [18] S. Capozziello, S. Carloni, A. Troisi, *Recent Res. Dev. Astron. Astrophys.* **1** 625 (2003).
- [19] S. Capozziello, V.F. Cardone, A. Troisi, *JCAP* **0608** 001 (2006).
- [20] S. Nojiri, S.D. Odintsov, *Int. J. Geom. Meth. Mod. Phys.* **4** 115 (2007).
- [21] C. Aktaş, S. Aygün, İ. Yılmaz, *Phys. Lett. B*, **707** 237 (2012).
- [22] T. Dereli, Ö. Sert, *Eur. Phys. J. C* **71** 3 1589 (2011).
- [23] T. Dereli, Ö. Sert, *Mod. Phys. Lett. A* **26** 1487 (2011).
- [24] Ö. Sert, "Gravity and Electromagnetism with $Y(R)F^2$ -type Coupling and Magnetic Monopole Solutions", arXiv:1203.0898.
- [25] A. R. Prasanna, *Phys. Lett.* **A37** 337 (1971).
- [26] G. W. Horndeski, *J. Math. Phys.* **17** 1980 (1976).
- [27] I. T. Drummond, S. J. Hathrell, *Phys. Rev. D* **22** 343 (1980).
- [28] H. A. Buchdahl, *J. Phys. A* **12** 1037 (1979).

- [29] T. Dereli, G. Üçoluk, *Class. Quant. Grav.* **7** 1109 (1990).
- [30] F. Müller-Hoissen, *Class. Quant. Grav.* **5** L35 (1988).
- [31] A. B. Balakin, J. P. S. Lemos, *Class. Quant. Grav.* **22** 1867 (2005).
- [32] A.B. Balakin, H. Dehnen, A.E. Zayats, *Phys. Rev. D* **79** 024007 (2009).
- [33] A.B. Balakin and A.E. Zayats, *Gravit. Cosmo.* **14** 86 (2008).
- [34] T. Dereli, Ö. Sert, *Phys. Rev. D* **83** 065005 (2011).
- [35] S. H. Mazharimousavi, M. Halilsoy, T. Tahamtan, *Eur. Phys. J. C* **72** 1851 (2012).
- [36] G. Lambiase, S. Mohanty, G. Scarpetta, *JCAP* **07** 019 (2008).
- [37] K. Bamba, S. Nojiri, S. D. Odintsov, *Phys. Rev. D* **77** 123532 (2008).
- [38] M. S. Turner, L. M. Widrow, *Phys. Rev. D* **37** 2743 (1988).
- [39] F. D. Mazzitelli, F. M. Spedalieri, *Phys. Rev. D* **52** 6694 (1995).
- [40] G. Lambiase, A. R. Prasanna, *Phys. Rev. D* **70** 063502 (2004).
- [41] A. Raya, J. E. M. Aguilar, M. Bellini, *Phys. Lett. B* **638** 314 (2006).
- [42] L. Campanelli, P. Cea, G. L. Fogli, L. Tedesco, *Phys. Rev. D* **77** 123002 (2008).
- [43] K-T. Kim, P. P. Kronberg, P. E. Dewdney, T.L. Landecker, *Astrophys. J.* **355** 29 (1990).
- [44] K-T. Kim, P. C. Tribble, P. P. Kronberg, *Astrophys. J.* **379** 80 (1991).
- [45] T. E. Clarke, P. P. Kronberg, H. Boehringer, *Astrophys. J.* **547** L111 (2001).
- [46] M. S. Turner, E. N. Parker, T.J. Bogdan, *Phys. Rev. D* **26** 6 (1982).
- [47] K. Bamba, S. D. Odintsov, *JCAP* **04** 024 (2008).
- [48] K. Bamba, S. Nojiri, S. D. Odintsov, *JCAP* **0810** 045 (2008).
- [49] M. R. Setare, J. Sadeghi, A. Banijamali, *Eur. Phys. J. C* **64** 433 (2009).
- [50] A. Banijamali, B. Fazlpour, *Eur. Phys. J. C* **71** 1684 (2011).
- [51] E. Garcia-Berro, J. Isern, Y. A. Kubyshin, *Astron. Astrophys. Rev.* **14** 113 (2007).
- [52] M. Visser, *Class. Quant. Grav.* **21** 2603 (2004).
- [53] D. Rapetti, S. W. Allen, M. A. Amin, R. D. Blandford *Mon. Not. R. Astron. Soc.* **375** 1510 (2007).

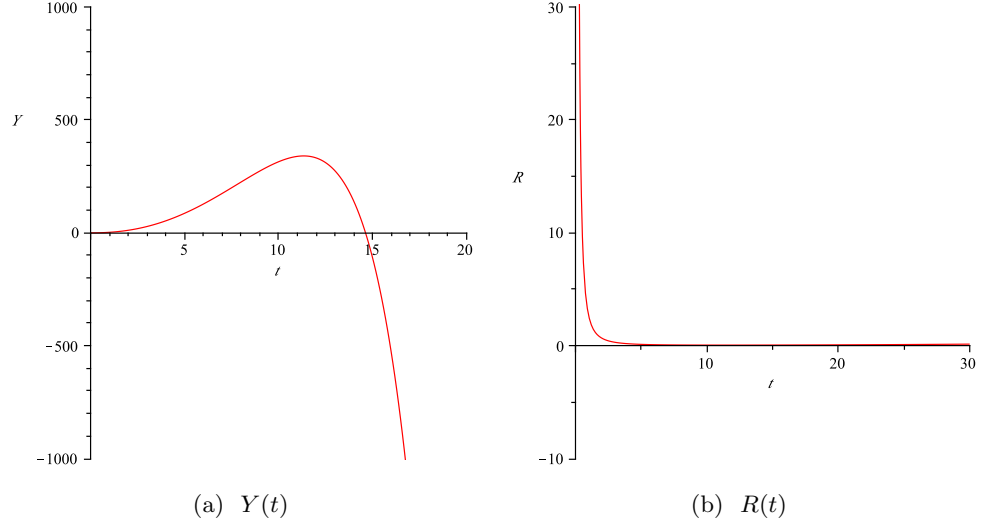


Figure 1: (a) The non-minimal coupling function and (b) the curvature scalar vs cosmic time t (Gyr) for $n = 1.063$, $\beta = 0.0000172$, $m = 3.053$, $B_0 = 1$ and $\kappa = 1$.

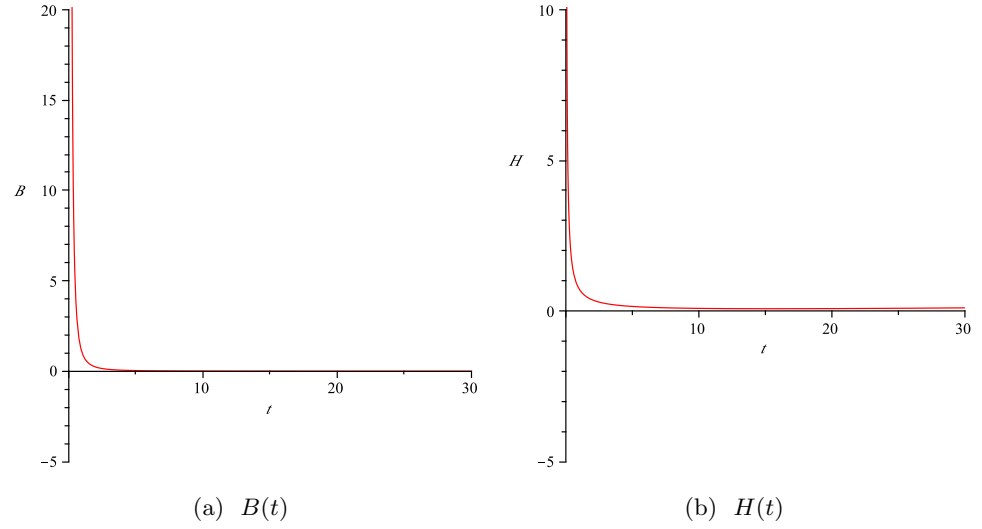


Figure 2: (a) The magnetic field and (b) the Hubble parameter vs cosmic time t (Gyr) for $n = 1.063$, $\beta = 0.0000172$, $m = 3.053$, $B_0 = 1$ and $\kappa = 1$.

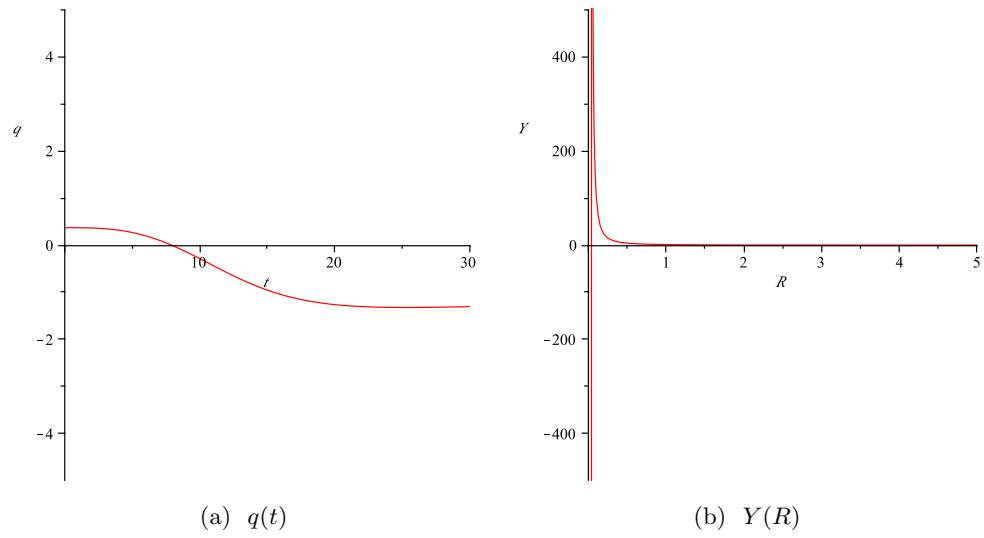


Figure 3: (a) The deceleration parameter and (b) the non-minimal function $Y(R)$ for $n = 1.063$, $\beta = 0.0000172$, $m = 3.053$, $B_0 = 1$ and $\kappa = 1$.